Dr. Barnali Dutta, Assistant Professor, Department of Mathematics,
Netaji Nagar Day College

Topic For

Semester-1, Paper-CC1(UG Hons.)

Graphical Demonstration of Different Type of Functions

1. Polynomial Function:

A polynomial function is a function that can be expressed in the form of a polynomial. A polynomial is generally represented as P(x). A polynomial function has only positive integers a exponents .The highest power of the variable of P(x) is known as its degree. Degree of a polynomial function is very important as it tells us about the behaviour of the function P(x) when x becomes very large.

- $x^2 + 2x + 1$
- $3x - 7$
- $7x^3 + x^2 - 2$

Theseare polynomial since all of the variables have positive integer exponents. But expressions like;

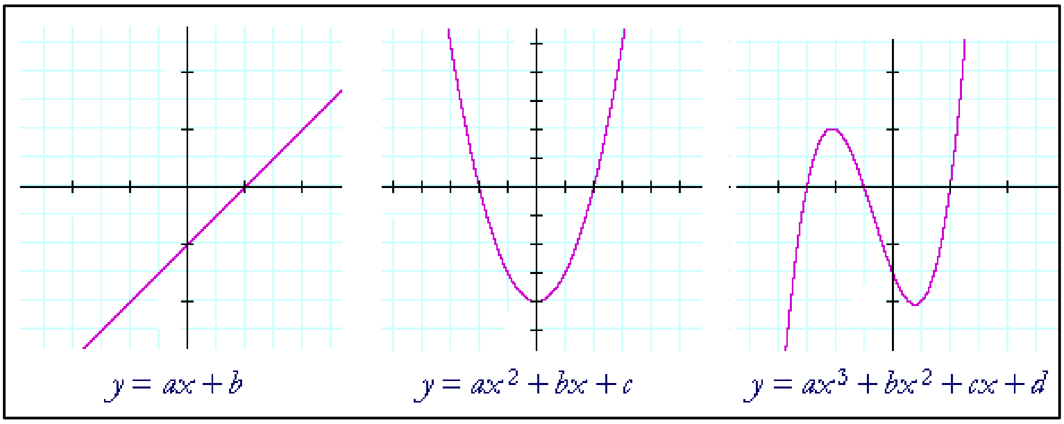
- $5x^{-1} + 1$
- $4x^{\frac{1}{2}} + 3x + 1$
- $(9x + 1) \div (x)$

are not polynomials, we cannot consider negative integer exponents or fraction exponent or division here.

The domain of a polynomial function is entire real numbers R. Polynomial are always continuous and differentiable.

There are various types of polynomial functions based on the degree of the polynomial. The most common types are:

- Linear Polynomial Function: $P(x) = ax + b$........a straight line
- Quadratic Polynomial Function: $P(x) = ax^2 + bx + c$......Parabola
- Cubic Polynomial Function: $ax^3 + bx^2 + cx + d$.....N-shaped



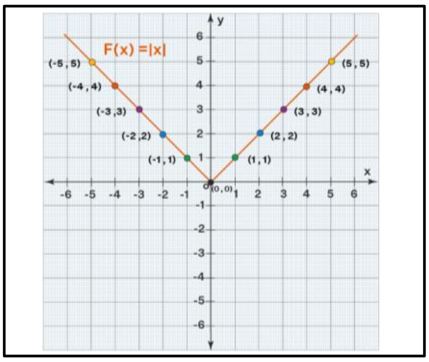
2. Modulus Function:

The modulus function, which is also called the absolute value of a function gives the magnitude or absolute value of a number irrespective of the number being positive or negative. It always gives a non-negative value of any number or variable. Modulus function is denoted as $y = |x| or\ f(x) = |x|$, where $f: R \rightarrow R\ and\ x \in R$ i.e.

$$f(x) = |x| = \begin{cases} x\ if\ x \geq 0 \\ -x\ if\ x < 0 \end{cases}$$

About the modulus function $f(x) = |x|$

- Domain: $x \in R$
- Range: $y \in [0, \infty)$
- Even function that means symmetry about y-axis.
- Many one function that means we get more than one pre image at least one image.
- Inverse does not exist since this is not a bijective mapping.
- Continuous function.
- Derivable at $x \in R - \{0\}$
- Into function
- Increasing in $x \in (0, \infty)$.
- Decreasing in $x \in (-\infty, 0)$.



$y = |x|$

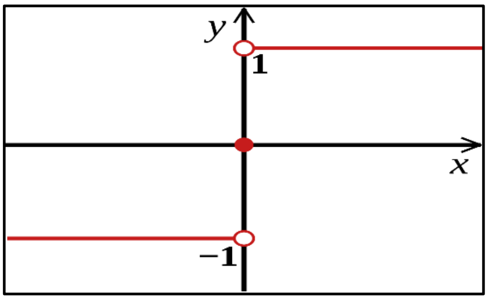
3. Signum Function:

The signum function simply gives the sign for the given values of x. For x value greater than zero, the value of the output is +1, for x value lesser than zero, the value of the output is -1, and for x value equal to zero, the output is equal to zero. The signum function can be defined and understood from the below expression.

$$sgn(x) = \begin{cases} 1 \ for\ x > 0 \\ -1\ for\ x < 0 \\ 0\ for\ x = 0 \end{cases}$$

About the signum function $f(x) = sgn(x)$:

- Domain:$x \in R$
- Range:$\{1,0,-1\}$
- Odd function i.e. symmetry about origin.
- Many one function.
- Inverse does not exist.
- Not continuous.
- Into function.



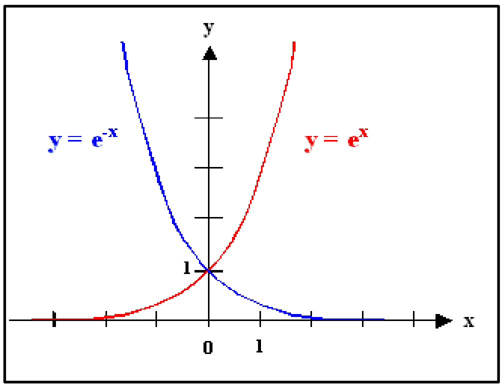
$y = sgn(x)$

4. Exponential Function:

An exponential function is a Mathematical function in form $f\ (x) = a^x$, where "x" is a variable and "a" is a constant which is called the base of the function and it should be greater than 0. The most commonly used exponential function base is the transcendental number e, which is approximately equal to 2.71828.

About the exponential function$y = e^x$:

- Domain: $x \in R$
- Range: $y \in R^+$
- One-one function
- Continuous
- Increasing when $a > 0$, decreasing when $a < 0$.
- The graph passes through the point (0,1).



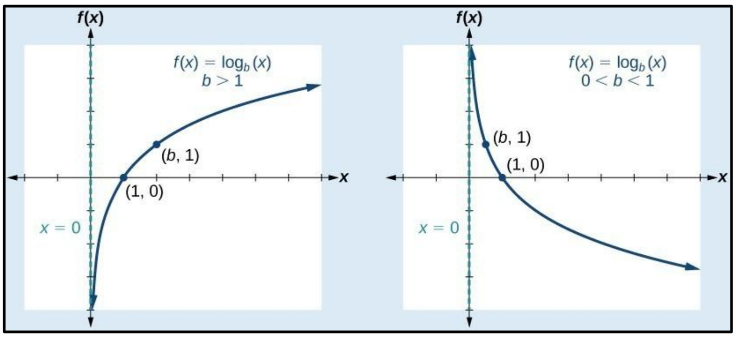
$$y = e^x$$

5. Logarithmic Function:

The *logarithm* of a positive real number x with respect to base b is the exponent by which b must be raised to yield x. In other words, the logarithm of x to base b is the unique real number y such that $b^y = x$ $i.e.$ $f(x) = y = log_b x$.

About Logarithmic Function $y = log_b x$:

- Domain: $x \in R^+$
- Range: $y \in R$
- Continuous
- One-to-one
- For $b > 1$ strictly increasing
- For $0 < b < 1$ strictly decreasing
- Bijective



$$y = log_b x$$

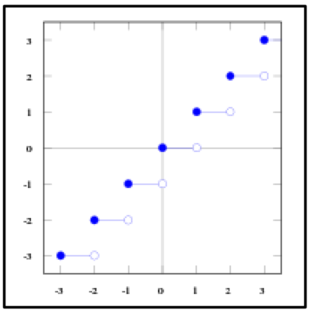
6. Greatest Integer Function $y = [x]$:

The greatest integer function (Floor function) is the function that takes as input a real number x and gives as output the greatest integer less than or equal to x, denoted $[x]$.

For example, $[2.4] = 2, [-2.4] = -3$.

About Greatest integer function $y = [x]$:

- Domain: $x \in R$
- Range: $y \in Z$
- Many-to -one
- Continuous everywhere except at integer
- Inverse does not exist



$$\boldsymbol{y = [x]}$$

7. <u>Fractional Part Function</u>:

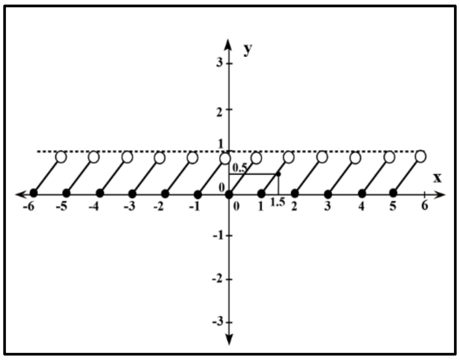
Let x be a real number. Then the fractional part of x is

$$\{x\} = x - [\mathrm{x}]$$

Example: $\{2.4\} = 2.4 - [2.4] = 2.4 - 2 = 0.4$

<u>About Fractional Part Function $y = \{x\}$:</u>

- Domain: $x \in R$
- Range: $y \in [0,1)$
- $0 \leq \{x\} < 1$ and $\{x\} = 0$ iff x is integer.
- $\{x\} + \{-x\} = \begin{cases} 0 \; if \; x \; is \; integer \\ 1 \quad otherwise \end{cases}$
- Not Continuous
- One-to-one function
- Increasing at $x \in [n, n+1), n \in Z$.
- Periodic Function with period 1.



$y = \{x\}$

8. Trigonometric Functions:

Trigonometric functions are also known as **Circular Functions** can be simply defined as the functions of an angle of a triangle. It means that the relationship between the angles and sides of a triangle are given by these trig functions. The basic trigonometric functions are sine, cosine, tangent, cotangent, secant and cosecant.

- The cos and sec functions are even functions; the rest other functions are odd .

$$\sin(-x) = -\sin x, \cos(-x) = \cos x, \tan(-x) = -\tan x, \cot(-x) = -\cot x, csc(-x) = -csc\ x, sec(-x) = sec\ x$$

- The trig functions are the periodic functions. The smallest periodic cycle is 2π but for tangent and the cotangent it is π.

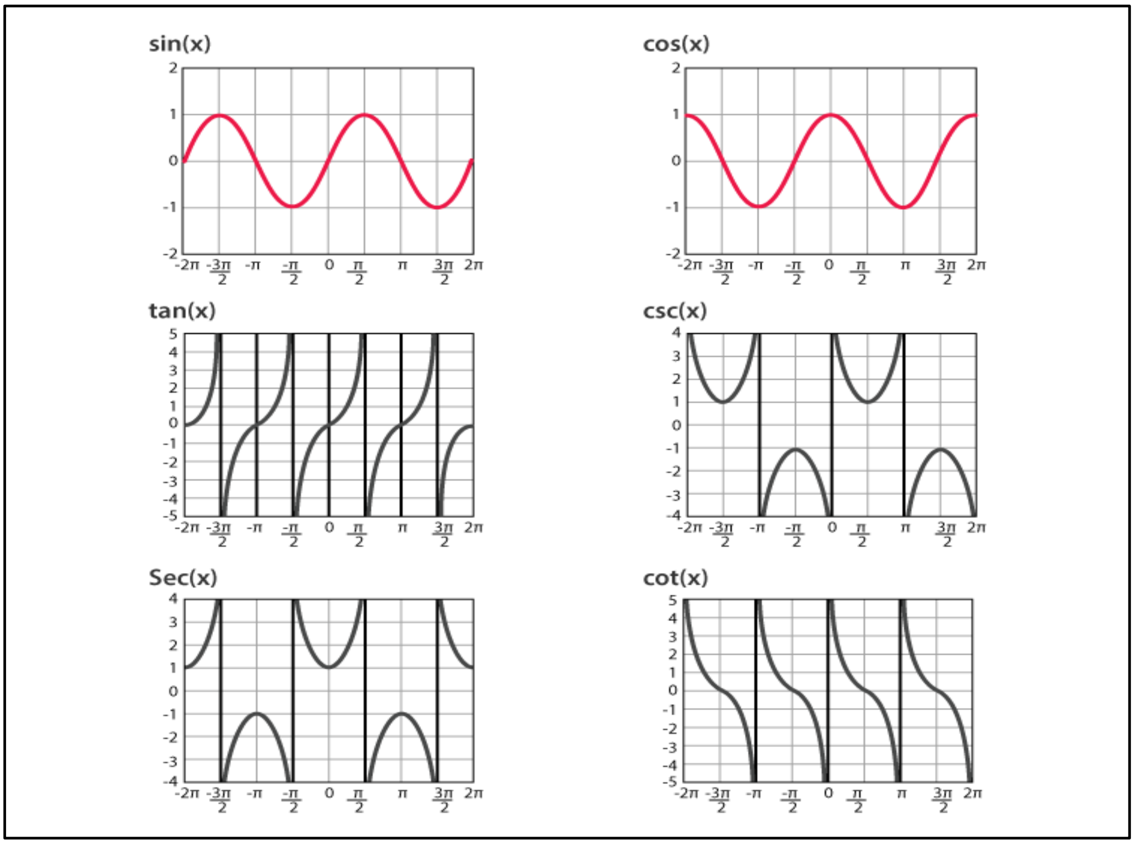
  $sin(x + 2n\pi) = sin\ x, cos(x + 2n\pi) = cos\ x, tan(x + n\pi) = tan\ x, cot(x + n\pi) = cot\ x, csc(x + 2n\pi) = csc\ x. sec(x + 2n\pi) = sec\ x$,

  Where n is any integer.

Before we see the graph, let us see the domain and range of each function, which is to begraphed in XY plane.

Function	**Definition**	**Domain**	**Range**
Sine Function	y=sin x	x ∈ R	− 1 ≤ sin x ≤ 1
Cosine Function	y = cos x	x ∈ R	− 1 ≤ cos x ≤ 1
Tangent Function	y = tan x	x ∈R , x≠(2k+1)π/2,	− ∞ < tan x < ∞
Cotangent Function	y = cot x	x ∈R , x ≠ k π	− ∞ < cot x < ∞
Secant Function	y = sec x	x ∈R , x ≠ (2 k + 1) π / 2	sec x ∈ (− ∞ , − 1] ∪ [1 , ∞)
Cosecant Function	y = csc x	x ∈R , x ≠ k π	csc x ∈ (− ∞ , − 1] ∪ [1 , ∞)

Here is the graph for all the functions based on their respective domain and range.


sin(x)
cos(x)
tan(x)
csc(x)
Sec(x)
cot(x)

****