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Topic for - Paper MTMA CC-10 (Mechanics)

Basic Concepts of Statics

1.1. Preliminary Concepts

It is natural for a student of physics or applied mathematics who endeavours to study the topics on classical mechanics, to think that this topic if treated as a branch of physics does not treat the new physical concepts which may lead him directly into current physical research or may help him in solving the practical problems of mechanics which are faced in the laboratory work. Classical Mechanics which was preliminary introduced as the study of motions of physical objects such as motion of celestial bodies is now considered as the part of mechanics dealing with the objects neither too big so that there exists a close agreement between theory and experiment, nor too small interacting object so that systems are considered on an atomic scale.

1.2. Mechanics

Mechanics can be defined as that science which describes and predicts the conditions of rest or motion of bodies around us under the action of forces where a body is said to be at rest when it does not change its position with and with reference to the surrounding objects but when it changes its position, we say that it is in motion.

Mechanics is divided into three parts: (i) mechanics of rigid bodies,
(ii) mechanics of deformable bodies, and (iii) mechanics of fluids.
The mechanics of rigid bodies is subdivided into three sub sections:

(i) **Kinematics**: Which deals with the all possible motions of material systems without reference to the agency which causes motion.

(ii) **Dynamics**: Which deals with the state of motion of bodies under the action of forces.

(iii) **Statics**: Which deals with the state of rest of bodies under the action of system of forces.

In this part of the study of mechanics, bodies are assumed to be perfectly rigid. Actual structures and machines, however, are never absolutely rigid and deform under the loads to which they are subjected. But these deformation are usually small and do not appreciably affect the conditions of equilibrium or motion of the structure under consideration. They are important, though, as far as the resistance of the structure to failure is concerned and are studied in mechanics of materials, which is a part of the mechanics of deformable bodies. The third division of mechanics, the mechanics of fluids, is subdivided into the study of incompressible fluids and of compressible fluids. An important sub division of the study of incompressible fluids is hydraulics, which deals with problems involving water.

Mechanics is a physical science, since it deals with the study of physical phenomena. However, some associate mechanics with mathematics, while many consider it as an engineering subject. Both these views are justified in part. Mechanics is the foundation of most engineering sciences and is an indispensible prerequisite to their study. However, it does not have the empiricism found in some engineering sciences, i.e., it does not rely on experience or observation alone; but its rigor and the emphasis it places on deductive reasoning it resembles mathematics. But, again, it is not an abstract or even a pure science; mechanics is an applied sciences. The purpose of mechanics is to explain and predict physical phenomena and thus to lay the foundations for applications.

1.3. Principles

The basic concepts used in mechanics are (i) Space, (ii) Time, (iii) Mass, and (iv) Force. These concepts cannot be truly defined; they should be accepted on the basis of our intuition and experience and used as a mental frame of reference for our study.

Space: The concept of space is associated with the notion of the position of a point. The position of a point can be defined by three lengths measured from a certain reference point, as origin, in three directions. These lengths are known as the co-ordinates of the point.

Time: To define an event, it is not sufficient to indicate its position in space. The time of the event should also be given.
Mass: The concept of mass is used to characterize and compare bodies on the basis of certain fundamental mechanical experiments. It is the quantity of matter packed in the body.

Force: A Force represents the action of one body on another. It can be exerted by actual contact or at a distance, as in the case gravitation forces and magnetic forces. A force is characterized by its point of application, its magnitude, and its direction; a force is represented by a vector.

Form of Forces: The forces may be classified into three different forms under which a force may appear when applied to a mass:

(i) **Tension and Thrust:** When a body is pulled by a string, a force of tension is exerted by the string on the body. When a body is pushed by anything, say by a rod, a force of thrust is exerted by the rod on the body.

(ii) **Action and Reaction:** When two bodies are in contact, each body exerts a force on the other which are equal in magnitude but opposite in direction. Either of these forces is called action and the other is called reaction.

(iii) **Attraction and Repulsion:** These are forces which have no visible contact with the body, e.g., gravity is the force of attraction of earth, repulsion is the force between two like poles of magnets etc.

(iv) **Friction:** Friction is a force exerted on bodies in contact due to relative motion of two bodies, one of them being rough.

(v) There are two types of friction: Sliding friction and Rolling friction depending upon the nature of the relative motion.

(vi) Weight: The weight of a body or particle is the force exerted by the earth to attract it towards the earth’s centre.

Two Important Terms

(i) **Equilibrium**: If a body be rest under the action of a system forces, the system of forces is said to be balanced and the body is said to be in equilibrium.

(ii) **Resultant**: If a system of forces can be replaced by a single force without disturbing the body's initial condition of rest of motion, then the single force is the resultant of the system of forces. Thus, a resultant is a single force capable of replacing the action of a system of forces on a body.

1.4. Fundamental Principles of Statics

All theorems and equations in mechanics, especially in statics are deduced from a few fundamental principles, which are accepted after experimental verification but without mathematical proof, and are known as the principles or axioms of statics.

First Principle: A free rigid body subject to the action of two forces can be in equilibrium if and only if, the two forces are equal in magnitude, collinear and opposite in direction.

Second Principle: The action of a given force system on a rigid body remains unchanged if another balanced force system is added to, or subtracted from, the original system.

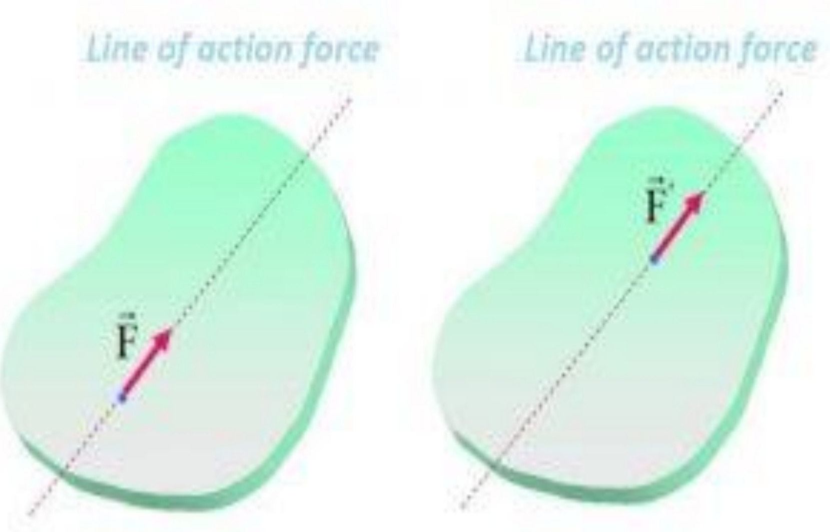
Third Principle: Two forces applied at one point of a body have as their resultant a force applied at the same point and represented by the diagonal of a parallelogram constructed with the two given forces as its sides.

Fourth Principle: To any action of one material body on another there is always an equal and opposite reaction, or, reaction is always equal to action.

Cor. From 1st and 2nd principles we can generalized as: the point of application of a force acting on a rigid body can be transferred to any other point on line of action of the force without altering its effect. It is known as Principle of Transmissibility.

1.5. The **principle of transmissibility** is applied to solid objects and affirms that a force applied on some point of the body is equivalent to another force of equal magnitude and direction, as long as said force is applied in the same line that contains the original force.

Therefore, any force of the same magnitude and direction will cause the same translational and rotational motion effect on the object, provided that its point of application is located on the same line, as shown in the following figure.



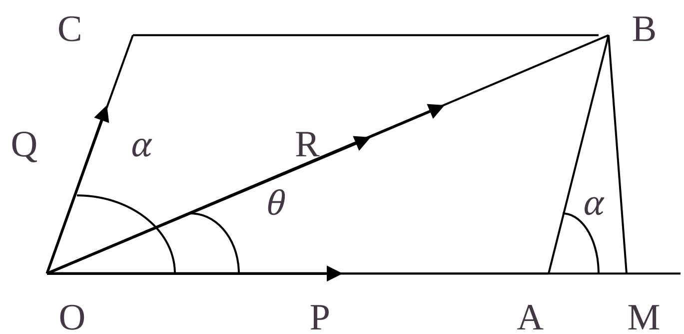
In a rigid body, the effect of a force remains unchanged when the point of application of the force moves along its line of action.

The forces shown **F** and **F′** are said to be equivalent forces and the dotted straight line that contains them is called the line of action of the force.

1.6. Composition and Resolution of Forces

Resultant: If two or more forces $F_1, F_2, F_3, ...$ act upon a rigid body and if a single force, R, can be found whose effect upon the body is the same as that of the forces $F_1, F_2, F_3, ...$ then this single force R is called the resultant of the given forces and the given forces $F_1, F_2, F_3, ...$ are called the components of R.

The **law of parallelogram of forces** states that if two vectors acting on a particle at the same time be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point .



Law of parallelogram of forces

This fundamental theorem of statics was first introduced by stevinus of Bruges in the year 1586.

From figure $OB^2 = OM^2 + BM^2$

$$= (OA + AM)^2 + BM^2$$

$$= (P + Qcos\alpha)^2 + (Qsin\alpha)^2$$

$$\therefore R^2 = P^2 + Q^2 + 2PQcos\alpha \quad \text{where } \alpha =< COA$$

Let θ be the inclination of R to OA

$$\therefore tan\theta = \frac{BM}{OM} = \frac{Qsin\alpha}{P + Qcos\alpha}$$

Note: 1. $= 0^o$, then R = P + Q and if $\alpha = 180^0$ then $R = P \sim Q$.

2. If $\alpha = 90^o$, $R = \sqrt{P^2 + Q^2}$, $Q = \tan^{-1}\left(\frac{Q}{P}\right)$.

3. If P = Q then $R = 2Pcos\left(\frac{\alpha}{2}\right)$ and $\theta = \frac{\alpha}{2}$.

4. The components of R are given by following relations:

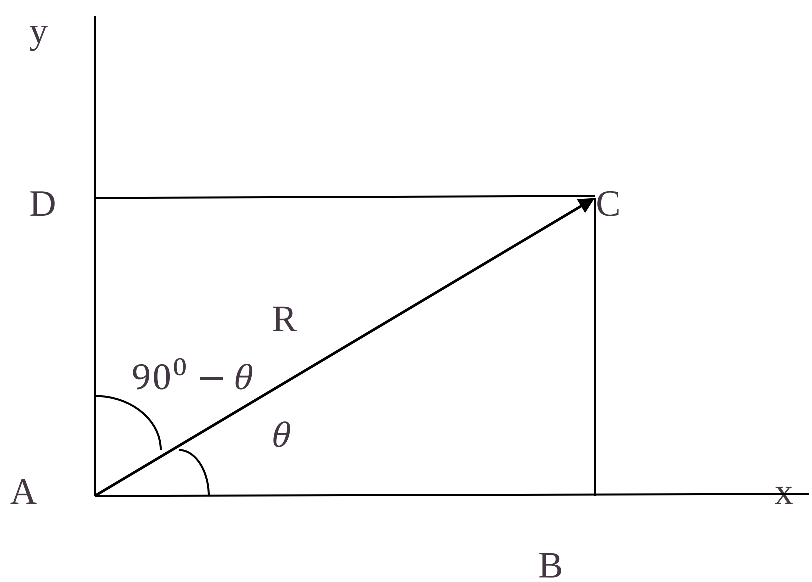
$$\frac{P}{sin(\alpha-\theta)}=\frac{Q}{sin\theta}=\frac{R}{sin(\pi-\alpha)}$$

$\therefore P = R\frac{sin(\alpha-\theta)}{sin\alpha}$ and $Q = R\frac{sin\theta}{sin\alpha}$.

If $\alpha = 90^0$ then $P = Rcos\theta$ and $Q = Rsin\theta$.

1.7. Rectangular Resolution:

Let R be the magnitude of a force represented by AC. Let Ax and Ay be two mutually perpendicular lines at A and θ the inclination of AC with Ax. Then components of R along Ax and Ay are AB and AD respectively.



$$\therefore \frac{AB}{AC} = cos\theta$$

i.e., $AB = ACcos\theta = Rcos\theta$

and $\frac{BC}{AC} = sin\theta$ i.e., $AD = BC = ACsin\theta = Rsin\theta = Rcos(90^0 - \theta)$

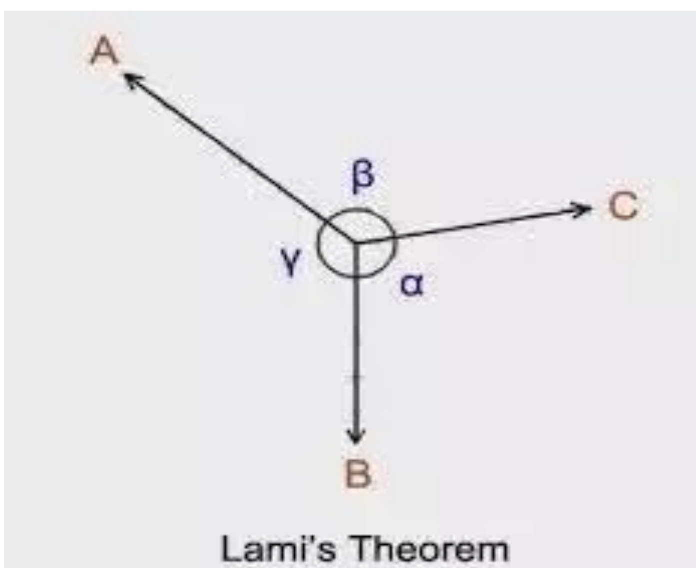
Thus, resolved part of a force in any direction = magnitude of the force × cosine of the angle between the given force and given direction.

Hence, the magnitude of R is equivalent to $Rcos\theta$ and $Rcos(90^0 - \theta)$ i.e., $Rsin\theta$ denoted by $R \equiv (Rcos\theta, Rsin\theta)$.

1.8. Lami's Theorem:

Lami's Theorem states, "When three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces". Referring to the above diagram, consider three forces A, B, C acting on a particle or rigid body making angles α, β and γ with each other.

$$\frac{A}{sin\alpha} = \frac{B}{sin\beta} = \frac{C}{sin\gamma}$$



Lami's Theorem

Note: Converse of the Lami's theorem is also true, i.e., if there forces acting at a point be such that each force is proportional to the sine of the angle between the other two, then the forces are in equilibrium.

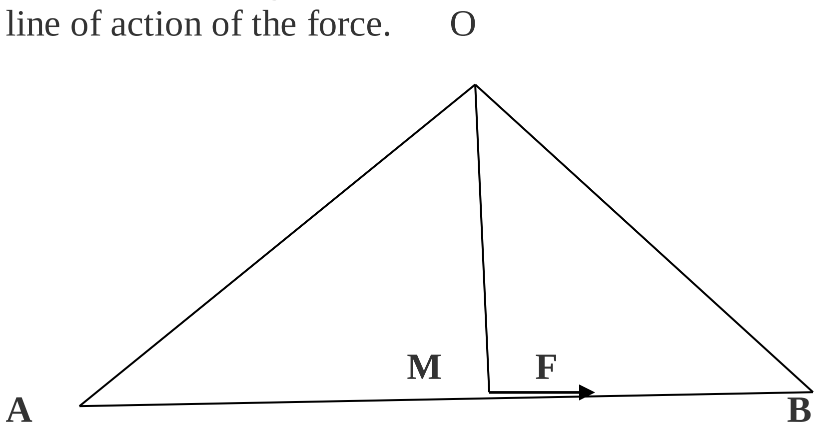
Polygon of Forces: If any number of forces acting at a point be represented in magnitude and direction by the slides of a closed polygon, taken in order, the forces will be in equilibrium.

The converse of the polygon of forces is not true; for the ratios of the sides of a polygon are not known when the directions of the sides are known.

1.9. Parallel Forces, Moments, Couples

In this section we shall discuss such type of forces which are parallel. In such cases, it is often necessary to find the resultant of two parallel forces. Now two parallel forces are said to be like when they act in the same sense i.e., direction and they are said to be unlike when they act in opposite senses i.e., in opposite directions.

Moments: The moment of a force about a given point is the product of the force and the length of perpendicular drawn from the given point upon the line of action of the force.



Thus the moment of F about a given point O = F$\times$ OM.
If the moment is zero, then either F is zero or length of perpendicular is zero i.e., the force passes through O.

If AB represents the force F, then the moment

$$= AB \times OM = \left(\frac{1}{2}AB.OM\right).2 = 2\Delta OAB$$

= twice the area of triangle OAB.

This is the geometrical meaning of the moment.

If the force has a tendency to turn the lamina about the fixed point in the counter –clockwise, the moment is taken as positive and if in the clockwise sense, the moment is negative.

Varignon's Theorem of Moments: If any number of co-planar forces acting on a rigid body have a resultant, the algebraic sum of their moments about any point in their plane is equal to the moment of the resultant about the same point.

Let $F_1, F_2, F_3, \ldots$ be the co-planar forces acting on a rigid body and $p_1, p_2, p_2, \ldots$ be the perpendicular distances from O on the line of action. Let R be the resultant and r be the distance from O. Then

$$F_1p_1 + F_2p_2 + F_3p_3 + \ \ldots . = Rr.$$

- If a system of coplanar forces be in equilibrium, i.e., if R = 0 then $\sum F_iP_i = 0.$
- If the line of action of the resultant R passes through O then also $\sum F_iP_i = 0.$
- If $\sum F_iP_i = 0$ and $R \neq 0$ then r=0 i.e., the point O lies on the line of action of the resultant.

Couples: Two equal unlike parallel forces, whose lines of action are not the same, form a couple.